Debate-based Learning Game for Constructing Mathematical Proofs

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Outline

Introduction

What is a debate?

Debate is a valuable and effective method of learning. It is an interactive process in which learners cooperate by exchanging arguments and counter-arguments to solve a common question.

- V. Durant-Guerrier, Argumentation and Proof in the Mathematics Classroom (In Proof and Proving in Mathematics education,2012).

- A. Pease, Budzynska K., Lawrence J., and C. Reed. Lakatos games for mathematical argument (COMMA, 2014)

- V. Durant-Guerrier, Argumentation and Proof in the Mathematics Classroom (In Proof and Proving in Mathematics education,2012).

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Our contribution

pragmatic tool for mathematics teaching and learning (mathematical proofs).

Argumentation Framework (Dung 1995)

An argumentation framework AF is a pair $\langle A, D \rangle$ where:

A: is a set of arguments.

$D \subseteq A \times A$: is a relation representing *defeat*

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Admissible Set

Given an $AF = \langle A, D \rangle$, a set $S \subseteq A$ is **admissible** in AF, if:

S is **conflict-free**

S **defends** all its elements.

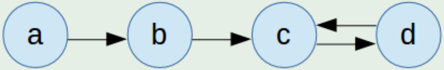
S defends all its elements if $\forall a \in S$, if $\exists b \in A \; s.t. (b, a) \in D$, then $\exists c \in S, s.t. (c, b) \in D$.

Grounded extension

Given an $AF = \langle A, D \rangle$, the grounded extension of AF is given by the least fixpoint of F_{AF}.

- $F_{AF} : 2^A \rightarrow 2^A$
- $F_{AF}(S) = \{a | a \text{ is acceptable with respect to } S\}$

Example



admissible(AF)= $\{\{a\}, \{d\}, \{a, d\}, \{a, c\}\}$

Grounded extension(AF)= $\{a, c\}$

Bipolar Argumentation Framework

A bipolar argumentation framework BAF is a triplet $\langle A, D, S \rangle$ where:

- A: is a set of arguments.
- $D \subseteq A \times A$: is a relation representing *defeat*
- $S \subseteq A \times A$: is a relation representing *support*

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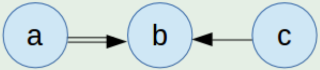


Definitions

Deductive argument

Let Γ be a set of formulas constructed from a given language L. An argument is a pair $\langle P, C \rangle$ where:

$$P \subseteq \Gamma$$

$$P \nvdash \bot$$

$$P \vdash_* C$$

$$P' \subseteq P, P' \vdash_* C$$

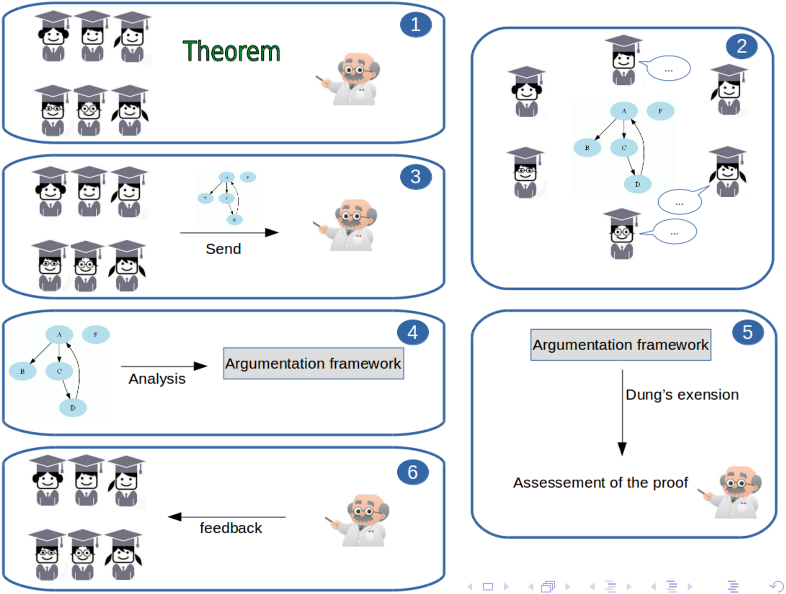
$\vdash_*$ is the inference symbol

Example

$$\langle \{n > 0, k > 0\}, k \times n > n \rangle$$

Debate-based learning game

process



Example

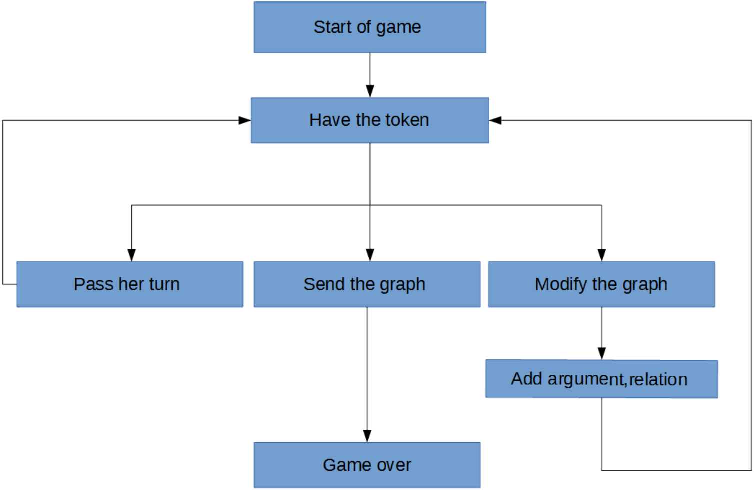
Prove if " $\forall \varepsilon > 0 \quad \exists N \in \mathbb{N}$ " then " $(n \geqslant N \Rightarrow 2 - \varepsilon < \dfrac{2n+1}{n+2} < 2 + \varepsilon)$. "

Example

- **Relations**=$\{$*support*, *defeat*$\}$
- **Propositions**:

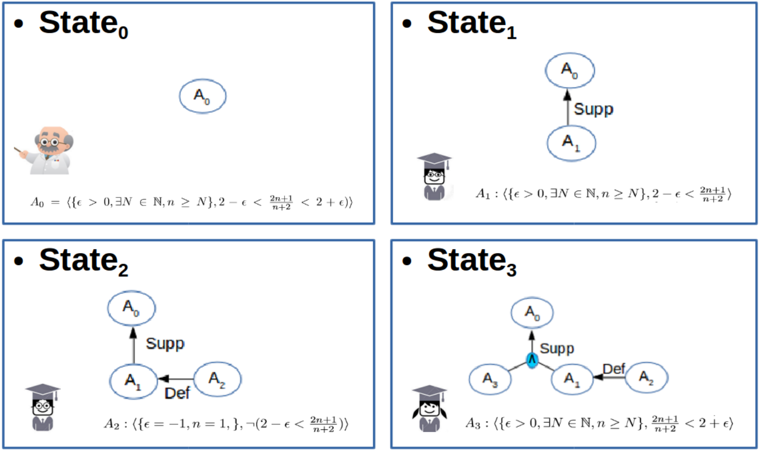
$$
\begin{array}{lllll}
\{n \in \mathbb{N}, & \frac{2n+1}{n+2} < 2, & \neg(\epsilon = -1), & \frac{2n+1}{n+2} < 2 + \epsilon, & \neg(\frac{2n+1}{n+2} < 2), \\
n \geq N, & \epsilon > 0, & N > \frac{3}{\epsilon} - 2, & n > \frac{3}{\epsilon} - 2, & \neg(N = [\frac{3}{\epsilon} - 2] + 1), \\
\epsilon = -1, & n = 1, & \neg(2 - \epsilon < \frac{2n+1}{n+2}), & n = -3, & \exists N \in \mathbb{N}, \\
\neg(n = -3), & (2-\epsilon)(n+2) < 2n+1\,, & & 2 - \epsilon < \frac{2n+1}{n+2} & \}.
\end{array}
$$

Example
Actions of the game



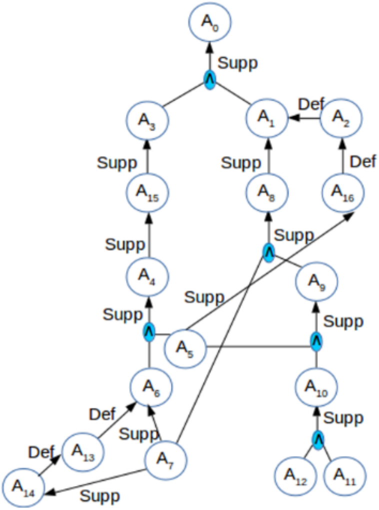
Example

Debate graph during the game



Example

Debate graph



$A_1 : \langle\{\epsilon > 0, \exists N \in \mathbb{N}, n \geq N\}, 2 - \epsilon < \frac{2n+1}{n+2}\rangle$

$A_2 : \langle\{\epsilon = -1, n = 1, \}, \neg(2 - \epsilon < \frac{2n+1}{n+2})\rangle$

$A_3 : \langle\{\epsilon > 0, \exists N \in \mathbb{N}, n \geq N\}, \frac{2n+1}{n+2} < 2 + \epsilon\rangle$

$A_4 : \langle\{\epsilon > 0, \frac{2n+1}{n+2} < 2\}, \frac{2n+1}{n+2} < 2 + \epsilon\rangle$

$A_5 : \langle\{\epsilon > 0\}, \epsilon > 0\rangle$

$A_6 : \langle\{n \in \mathbb{N}\}, \frac{2n+1}{n+2} < 2\rangle$

$A_7 : \langle\{n \in \mathbb{N}\}, n \in \mathbb{N}\rangle$

$A_8 : \langle\{n \in \mathbb{N}, (2 - \epsilon)(n + 2) < 2n + 1\}, 2 - \epsilon < \frac{2n+1}{n+2}\rangle$

$A_9 : \langle\{\epsilon > 0, n > \frac{3}{\epsilon} - 2\}, (2 - \epsilon)(n + 2) < 2n + 1\rangle$

$A_{10} : \langle\{n \geq N, N > \frac{3}{\epsilon} - 2\}, n > \frac{3}{\epsilon} - 2\rangle$

$A_{11} : \langle\{N > \frac{3}{\epsilon} - 2\}, N > \frac{3}{\epsilon} - 2\rangle$

$A_{12} : \langle\{n \geq N\}, n \geq N\rangle$

$A_{13} : \langle\{n = -3\}, \neg(\frac{2n+1}{n+2} < 2)\rangle$

$A_{14} : \langle\{n \in \mathbb{N}\}, \neg(n = -3)\rangle$

$A_{15} : \langle\{n \in \mathbb{N}, \epsilon > 0\}, \frac{2n+1}{n+2} < 2 + \epsilon\rangle$

$A_{16} : \langle\{\epsilon > 0\}, \neg(\epsilon = -1)\rangle$

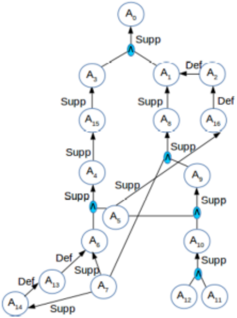
Example

Debate graph

Send the debate graph

Example

Analysis



Arguments

Defeat relation

Support relation

Example

Analysis

Bipolar Argumentation Framework

$BAF = \langle \mathcal{A}, \mathcal{D}, \mathcal{S} \rangle$ where:

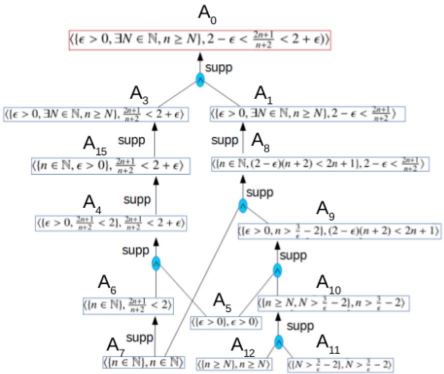
- $\mathcal{A} = \{A_0, A_1 \wedge A_3, A_2, A_4, A_5 \wedge A_6\ , A_7 \wedge A_9, A_8, A_5 \wedge A_{10}, A_{11} \wedge A_{12}\ , A_{13}, A_{14}, A_{15}, A_{16}\},$
- $\mathcal{D} = \{(A_2, A_1), (A_{16}, A_2), (A_{13}, A_6), (A_{14}, A_{13})\}$
- $\mathcal{S} = \{(A_1 \wedge A_3, A_0), (A_8, A_1), (A_9 \wedge A_7, A_8), (A_{10} \wedge A_5, A_9), (A_{11} \wedge A_{12}, A_{10}), (A_{15}, A_3), (A_4, A_{15}), (A_5 \wedge A_6, A_4), (A_7, A_6), (A_5, A_{16}), (A_7, A_{14})\}$

Grounded extension

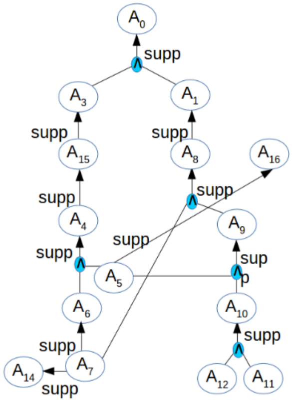
$GE = \{A_0, A_1 \wedge A_3, A_8, A_7 \wedge A_9, A_5 \wedge A_{10}, A_{11} \wedge A_{12}, A_{15}, A_4, A_6 \wedge A_5\ , A_{14}, A_{16}\}$

Example

Assessement



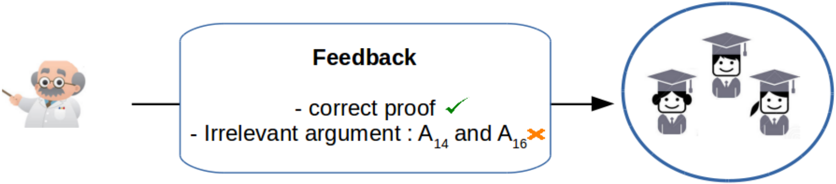
Graph of the proof



Debate graph analyzed

Example

Instructor feedback



Conclusion and Perspectives

- Debate-based learning game: mathematics domain, argumentation theory and game theory.

- The principle of the game has been tested by a group of learners and instructors.

- The system is acceptable to learners and instructors.

- Extension of the current work with preferences among learners.

- Finalize our system.

THANK YOU FOR YOUR ATTENTION