The Complexity of Inferences and Explanations in Probabilistic Logic Programming

Fabio G. Cozman, Denis D. Mauá
Universidade de São Paulo

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Overview

1. Probabilistic disjunctive logic programming.
2. The complexity of inferences and explanations.

Probabilistic disjunctive logic programs

- A probabilistic disjunctive logic program is a pair $\langle \mathbf{P}, \mathbf{PF} \rangle$:
  - **P** is a disjunctive logic program (no functions) and
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- Predicate r, atom $\mathrm{r}(\mathrm{t}_1, \ldots, \mathrm{t}_k)$, rule

$$A_1 \vee \cdots \vee A_h :- B_1, \ldots, B_{b'}, \textbf{not } C_{b'+1}, \ldots, \textbf{not } C_b$$

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  - A program without disjunction is *normal*.
  - A program without logical variables is *propositional*.

Probabilistic facts

- A probabilistic fact is a fact associated with a probability:

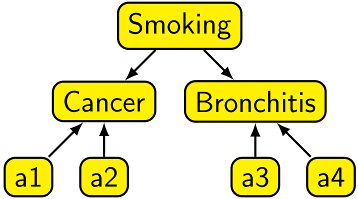
$$\mathbb{P}(A) = \alpha.$$

- Probabilistic facts are assumed independent.

Example: the Bayesian network Asia

- Predicates smoking, cancer, and bronchitis.
- Probabilistic logic program (ProbLog notation):

0.5 :: smoking.
cancer :− smoking, a1.
cancer :− **not** smoking, a2.
bronchitis :− smoking, a3.
bronchitis :− **not** smoking, a4.
0.1 :: a1.   0.01 :: a2.
0.6 :: a3.   0.3 :: a4.



Stratified normal programs:

- ... the grounded dependency graph has no cycle containing a *negative* edge.

- Example:

$$\text{path}(X, Y) :- \text{edge}(X, Y).$$
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$$0.6 :: \text{edge}(1, 2). \quad 0.1 :: \text{edge}(1, 3).$$
$$0.4 :: \text{edge}(2, 5). \quad 0.3 :: \text{edge}(2, 6).$$
$$0.3 :: \text{edge}(3, 4). \quad 0.8 :: \text{edge}(4, 5).$$
$$0.2 :: \text{edge}(5, 6).$$

A word on semantics

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Stable models:

- Consider logic program **P**.
- For some interpretation $\mathcal{I}$, take the reduct $\mathbf{P}^{\mathcal{I}}$:
  - Ground **P**.
  - Remove rules with subgoal **not** A and $A \in \mathcal{I}$.
  - Remove subgoals **not** A from remaining rules.
- Interpretation $\mathcal{I}$ is stable model if $\mathcal{I}$ is the minimal model of $\mathbf{P}^{\mathcal{I}}$.

Non-stratified program (cycle with negative edge)

- Non-stratified program may have more than one stable model.

The Dilbert example

$$\text{single}(X) \coloneq \text{man}(X), \textbf{not } \text{husband}(X).$$

$$\text{husband}(X) \coloneq \text{man}(X), \textbf{not } \text{single}(X).$$

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- man(dilbert) is false: a unique stable model s_1.
- man(dilbert) is true: there are two stable models,

$$s_2 = \{\text{husband}(\text{dilbert}) = \text{true}, \text{single}(\text{dilbert}) = \text{false}\},$$

and

$$s_3 = \{\text{husband}(\text{dilbert}) = \text{false}, \text{single}(\text{dilbert}) = \text{true}\}.$$

What could be the semantics of a non-stratified program?

- Probabilities over well-founded models:
  - Sato, Kameya and Zhou (2005),
  - Hadjichristodolou and Warren (2012).
  - Riguzzi (2015).

- Proposal by Lukasiewicz (2005):
  informally, take the set of every possible probability distributions that satisfy the rules and (probabilistic) facts.
  - We adopt name *credal semantics*.
  - Note: another recent semantics based on credal sets by Michels et al. (2015).

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- Take any $\gamma \in [0, 1]$:

$$\mathbb{P}(s_1) = 0.1, \quad \mathbb{P}(s_2) = 0.9\gamma, \quad \mathbb{P}(s_3) = 0.9(1 - \gamma).$$

An example: robot navigation (graph coloring...)

$$\text{color}(X, \text{red}) \vee \text{color}(X, \text{green}) \vee \text{color}(X, \text{yellow}) :- \text{site}(X).$$

$$\text{clash} :- \textbf{not}\ \text{clash}, \text{edge}(X, Y), \text{color}(X, C), \text{color}(Y, C).$$

$$\text{path}(X, Y) :- \text{edge}(X, Y). \qquad \text{path}(X, Y) :- \text{edge}(X, Z), \text{path}(Z, Y).$$

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$$\mathrm{site}(1). \quad \mathrm{site}(2). \quad \mathrm{site}(3). \quad \mathrm{site}(4). \quad \mathrm{site}(5).$$

$$\mathrm{color}(2,\mathrm{red}). \quad \mathrm{color}(5,\mathrm{green}).$$

$$0.5 :: \mathrm{edge}(4,5).$$

$$\mathrm{edge}(1,3). \quad \mathrm{edge}(1,4). \quad \mathrm{edge}(2,1). \quad \mathrm{edge}(2,4). \quad \mathrm{edge}(3,5). \quad \mathrm{edge}(4,3).$$

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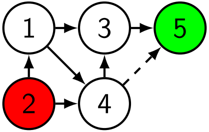

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- Inference: whether $\mathbb{P}(\mathbf{Q}|\mathbf{E}) > \gamma$.
- MPE: whether there is an interpretation $\mathbf{Q}$ that agrees with literals $\mathbf{E}$, such that $\mathbb{P}(\mathbf{Q}) > \gamma$.
- MAP: whether there is a partial interpretation $\mathbf{Q}$ that agrees with literals $\mathbf{E}$, such that $\mathbb{P}(\mathbf{Q}|\mathbf{E}) > \gamma$.

Results

	Propositional			Bounded arity		
	Inferential	MPE	MAP	Inferential	MPE	MAP
Acyclic normal	PP	NP	NP^{PP}	PP^{NP}	Σ_2^P	NP^{PP}
No negation, normal	PP	NP	NP^{PP}	PP^{NP}	Σ_2^P	NP^{PP}
Stratified normal	PP	NP	NP^{PP}	PP^{NP}	Σ_2^P	NP^{PP}
Normal, credal	PP^{NP}	Σ_2^P	NP^{PP}	$PP^{\Sigma_2^P}$	Σ_3^P	NP^{PP}
Normal, well-founded	PP	NP	NP^{PP}	PP^{NP}	Σ_2^P	NP^{PP}
Disjunctive, credal	$PP^{\Sigma_2^P}$	Σ_3^P	NP^{PP}	$PP^{\Sigma_3^P}$	Σ_4^P	NP^{PP}

(Complexity class $\Sigma_i^P = NP^{\Sigma_{i-1}^P}$.)
(Complexity class PP: class of problems solved by a probabilistic polynomial-time Turing machine.)

(In orange: PGM2016, WPLP2016, ENIAC2016.)

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Conclusion

- Main goal was to map the complexity of probabilistic disjunctive logic programming (and its sub-languages) and credal and well-founded semantics.
- Future work: remove bounds on arity, and consider query complexity.

- Thanks to support by CNPq and FAPESP.