Independent Natural Extension for Infinite Spaces

Williams-coherence to the Rescue!

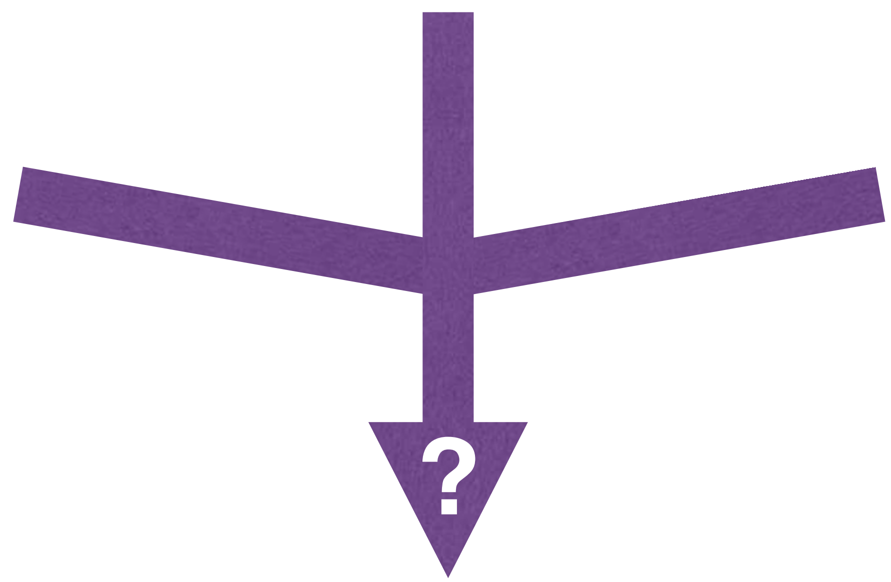
ECSQARU 2017
ISIPTA '17

Jasper De Bock

Ghent University
Belgium

X_1 **independent** X_2

local uncertainty model **local uncertainty model**



joint uncertainty model

$$P(X_1|X_2) = P(X_1)$$
$$P(X_2|X_1) = P(X_2)$$

X_1

independent

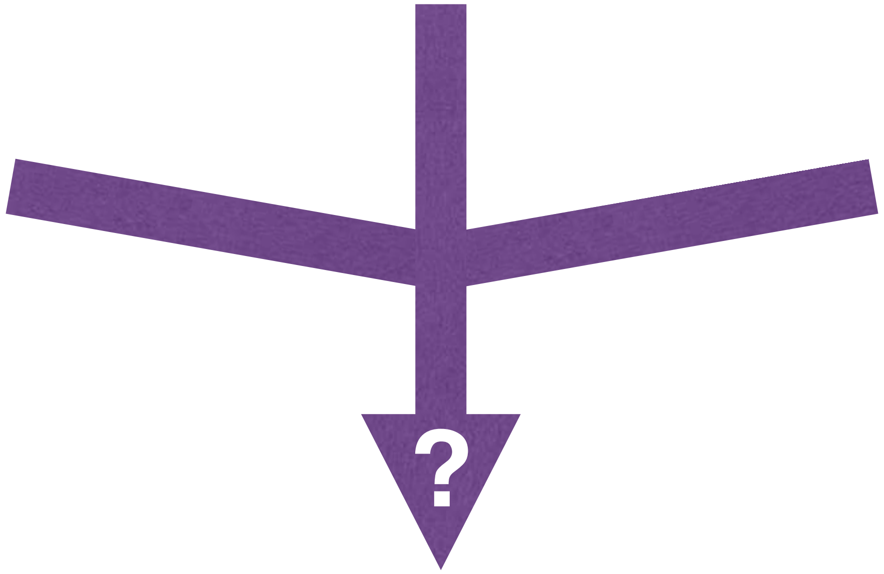
X_2

local uncertainty model

$P(X_1)$

local uncertainty model

$P(X_2)$



joint uncertainty model

$$P(X_1, X_2)$$

$$P(X_1|X_2) = P(X_1)$$
$$P(X_2|X_1) = P(X_2)$$

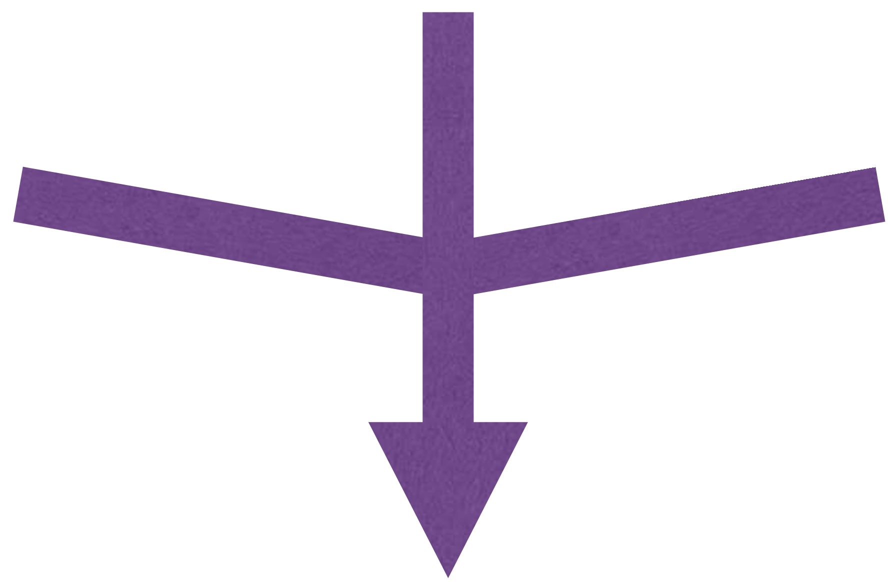
$$X_1$$

independent

$$X_2$$

local uncertainty model

$$P(X_1)$$



local uncertainty model

$$P(X_2)$$

joint uncertainty model

$$P(X_1, X_2) = P(X_1)P(X_2)$$

$$X_1$$

local uncertainty model

$$P(X_1),\ \underline{P}(X_1),\ \mathcal{D}_1,\ P(f(X_1)),\ \underline{P}(f(X_1)),\ \ldots$$

$$E(f(X_1)) \quad \underline{E}(f(X_1))$$

$$X_1$$

local uncertainty model

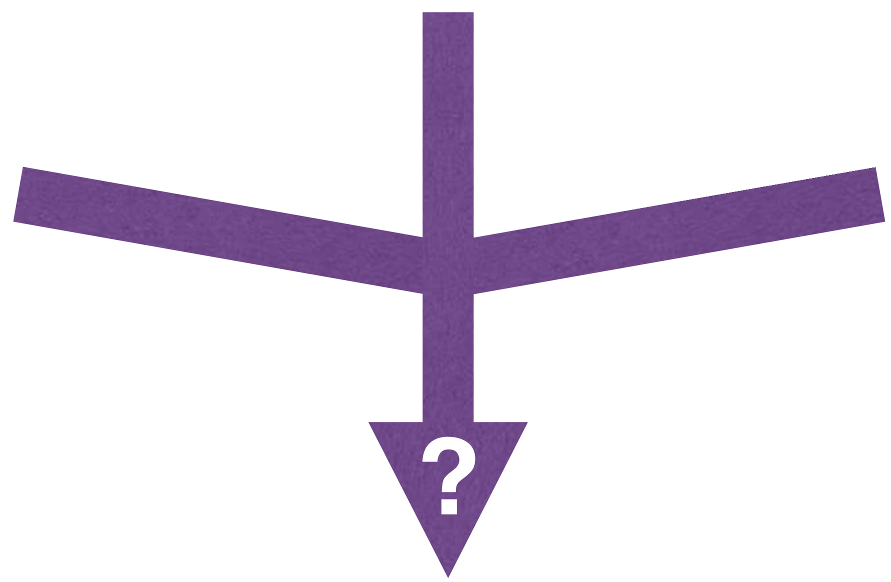
$$P(X_1),\ \underline{P}(X_1),\ \mathcal{D}_1,\ P(f(X_1)),\ \underline{P}(f(X_1)),\ \ldots$$

$$E(f(X_1)) \quad \underline{E}(f(X_1))$$

X_1 **independent** X_2

local uncertainty model

$\underline{P}(f(X_1))$



local uncertainty model

$\underline{P}(f(X_2))$

joint uncertainty model

$\underline{P}(f(X_1, X_2))$

X_1

local
uncertainty
model

$\underline{P}(f(X_1))$

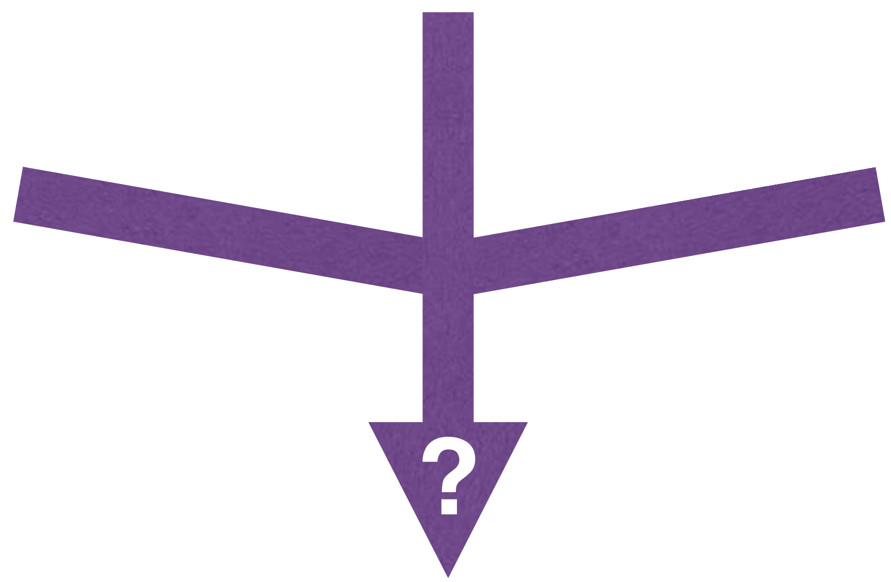
?

independent

X_2

local
uncertainty
model

$\underline{P}(f(X_2))$



joint uncertainty model

$\underline{P}(f(X_1, X_2))$

$$\underline{P}(f(X_1)|X_2) = \underline{P}(f(X_1))$$
$$\underline{P}(f(X_2)|X_1) = \underline{P}(f(X_2))$$

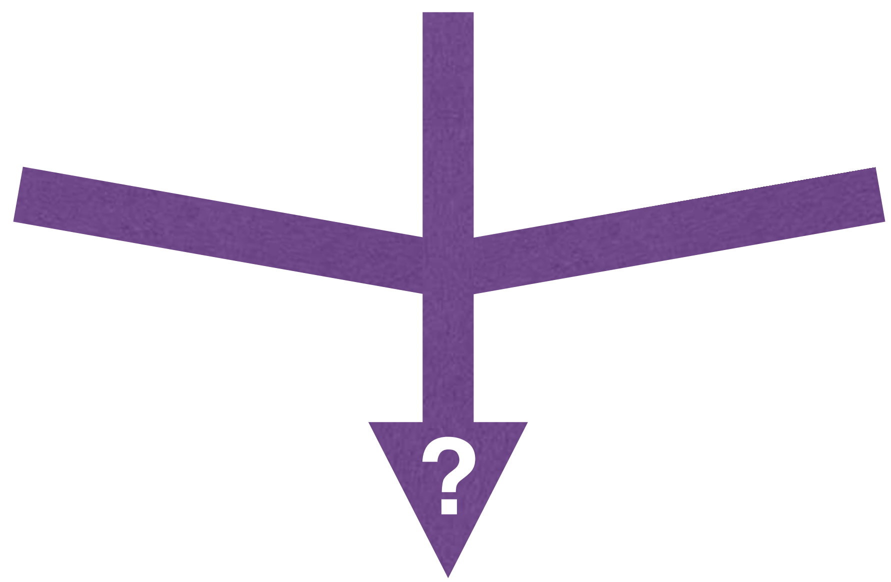
X_1

independent

X_2

local uncertainty model

$\underline{P}(f(X_1))$



local uncertainty model

$\underline{P}(f(X_2))$

joint uncertainty model

$$\underline{P}(f(X_1, X_2))$$

$$\underline{P}(f(X_1)|X_2) = \underline{P}(f(X_1))$$
$$\underline{P}(f(X_2)|X_1) = \underline{P}(f(X_2))$$

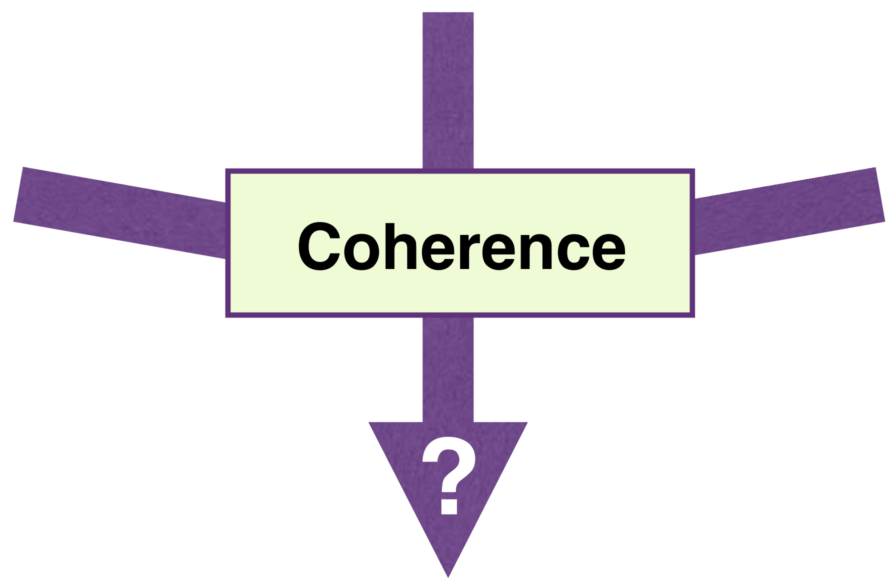
X_1 **independent** X_2

local uncertainty model

$\underline{P}(f(X_1))$

local uncertainty model

$\underline{P}(f(X_2))$



joint uncertainty model

$$\underline{P}(f(X_1, X_2))$$

$$\underline{P}(f(X_1)|X_2) = \underline{P}(f(X_1))$$
$$\underline{P}(f(X_2)|X_1) = \underline{P}(f(X_2))$$

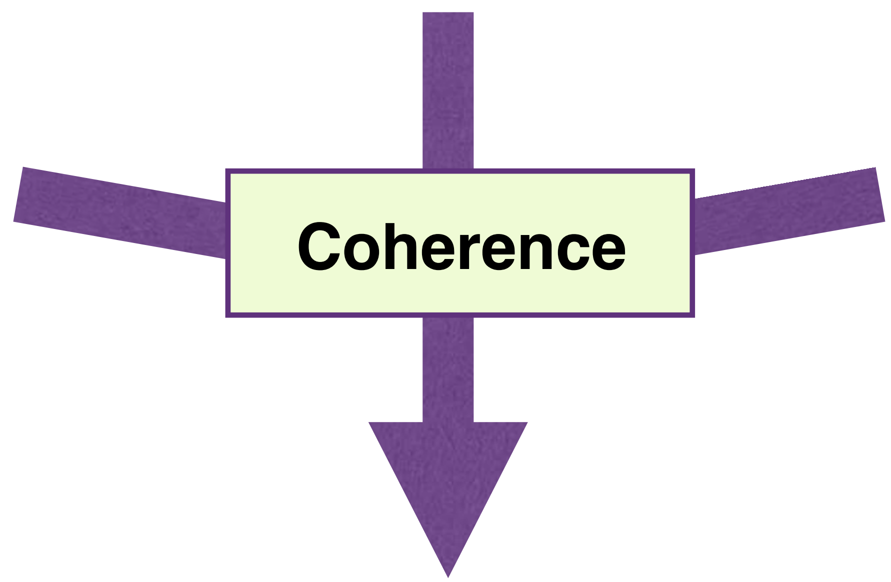
X_1 **independent** X_2

local uncertainty model

$$\underline{P}(f(X_1)) = \underline{P}_1(f)$$

local uncertainty model

$$\underline{P}(f(X_2)) = \underline{P}_2(f)$$



joint uncertainty model

$$(\underline{P}_1 \otimes \underline{P}_2)(f(X_1, X_2))$$

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Two very useful properties

External additivity

$$(\underline{P}_1 \otimes \underline{P}_2)(f(X_1) + h(X_2)) = \underline{P}_1(f(X_1)) + \underline{P}_2(h(X_2))$$

Factorisation

$$(\underline{P}_1 \otimes \underline{P}_2)(g(X_1)h(X_2))$$

$$= \begin{cases} \underline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \geq 0 \\ \overline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \leq 0 \end{cases}$$

if $g \geq 0$

DISCLAIMER!

All of this is well known, and has been for several years now…

DISCLAIMER!

All of this is well known, and has been for several years now…

…but only for finite spaces!

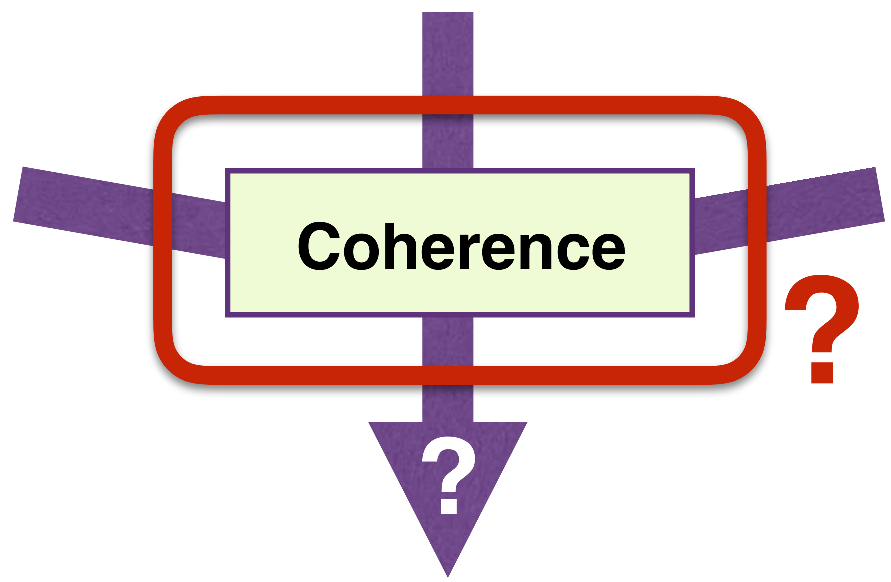
Independent Natural Extension for Infinite Spaces

$$\underline{P}(f(X_1)|X_2) = \underline{P}(f(X_1))$$
$$\underline{P}(f(X_2)|X_1) = \underline{P}(f(X_2))$$

X_1 **independent** X_2

local uncertainty model

$\underline{P}(f(X_1))$



local uncertainty model

$\underline{P}(f(X_2))$

joint uncertainty model

Coherence ?

Walley ⟷ Williams

Independent natural extension may not exist!


Independent products in infinite spaces

Miranda & Zaffalon 2015

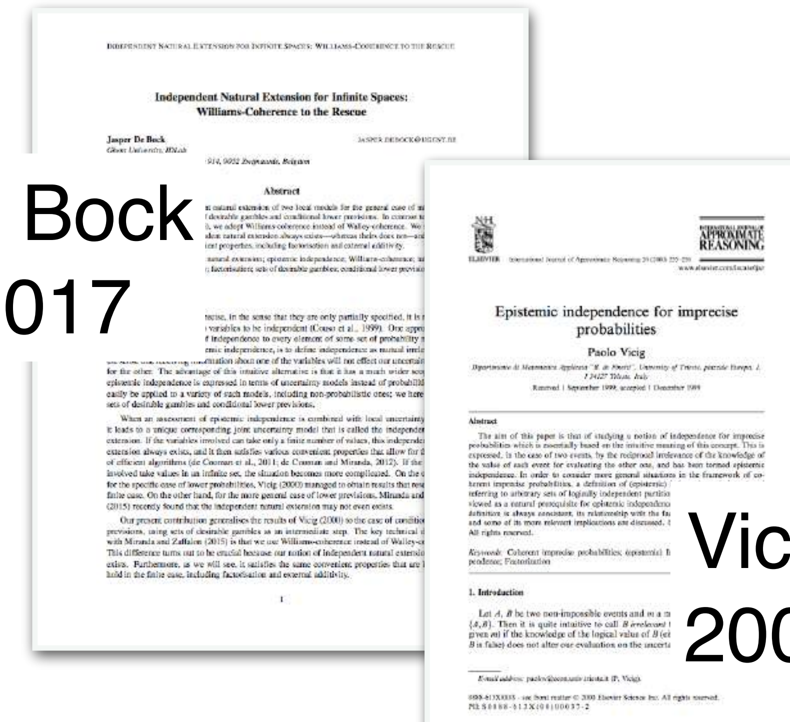
Coherence ?

~~Walley~~ Williams

Independent natural extension may not exist!

Independent natural extension always exists!

Miranda & Zaffalon 2015



De Bock 2017

Vicig 2000

Coherence

~~Walley~~ ⟷ Williams ✓

Independent Natural Extension for Infinite Spaces

Williams-coherence to the Rescue!

Two very useful properties

External additivity ?

$$(\underline{P}_1 \otimes \underline{P}_2)(f(X_1) + h(X_2)) = \underline{P}_1(f(X_1)) + \underline{P}_2(h(X_2))$$

Factorisation ?

$$(\underline{P}_1 \otimes \underline{P}_2)(g(X_1)h(X_2))$$
$$= \begin{cases} \underline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \geq 0 \\ \overline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \leq 0 \end{cases}$$

$$\text{if } g \geq 0$$

Two very useful properties

External additivity ✓

$$(\underline{P}_1 \otimes \underline{P}_2)(f(X_1) + h(X_2)) = \underline{P}_1(f(X_1)) + \underline{P}_2(h(X_2))$$

Factorisation ?

$$(\underline{P}_1 \otimes \underline{P}_2)(g(X_1)h(X_2))$$

$$= \begin{cases} \underline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \geq 0 \\ \overline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \leq 0 \end{cases}$$

if $g \geq 0$

$$\underline{P}(f(X_1)|X_2) = \underline{P}(f(X_1))$$
$$\underline{P}(f(X_2)|X_1) = \underline{P}(f(X_2))$$

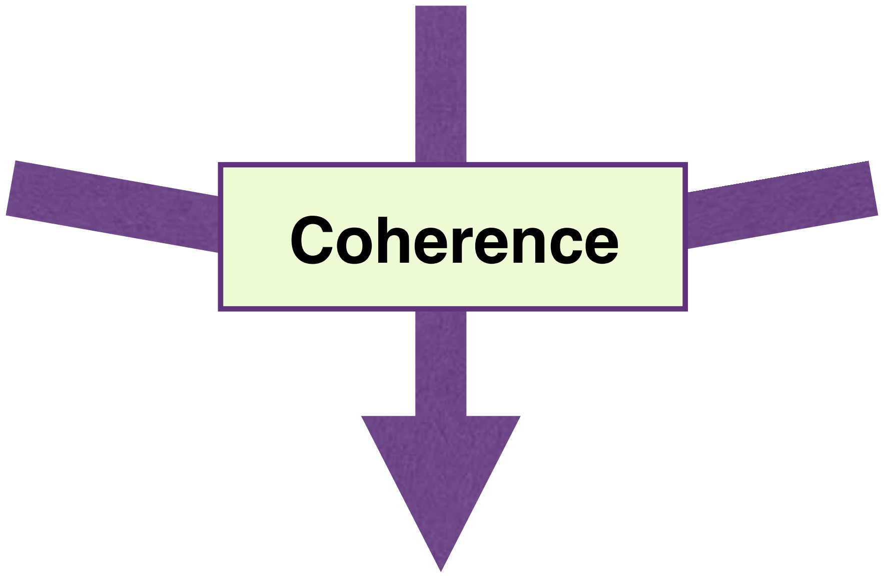
X_1 **independent** X_2

local uncertainty model

$$\underline{P}(f(X_1)) \\ || \\ \underline{P}_1(f)$$

local uncertainty model

$$\underline{P}(f(X_2)) \\ || \\ \underline{P}_2(f)$$



joint uncertainty model

$$(\underline{P}_1 \otimes \underline{P}_2)(f(X_1, X_2))$$

$$\underline{P}(f(X_1)|X_2) = \underline{P}(f(X_1))$$
$$\underline{P}(f(X_2)|X_1) = \underline{P}(f(X_2))$$

independent

$$\underline{P}(f(X_1)|B_2) = \underline{P}(f(X_1)) \quad \forall B_2 \in \mathcal{B}_2$$
$$\underline{P}(f(X_2)|B_1) = \underline{P}(f(X_2)) \quad \forall B_1 \in \mathcal{B}_1$$

$$\underline{P}(f(X_1)|X_2) = \underline{P}(f(X_1))$$
$$\underline{P}(f(X_2)|X_1) = \underline{P}(f(X_2))$$

independent

$$\underline{P}(f(X_1)|B_2) = \underline{P}(f(X_1)) \quad \forall B_2 \in \mathcal{B}_2$$
$$\underline{P}(f(X_2)|B_1) = \underline{P}(f(X_2)) \quad \forall B_1 \in \mathcal{B}_1$$

value-independence: $\mathcal{B}_i = \{\{x_i\} \colon x_i \in \mathcal{X}_i\}$

subset-independence: $\mathcal{B}_i = \mathcal{P}(\mathcal{X}_i) \setminus \{\emptyset\}$

Two very useful properties

External additivity ✓

$$(\underline{P}_1 \otimes \underline{P}_2)(f(X_1) + h(X_2)) = \underline{P}_1(f(X_1)) + \underline{P}_2(h(X_2))$$

Factorisation ?

$$(\underline{P}_1 \otimes \underline{P}_2)(g(X_1)h(X_2))$$

$$= \begin{cases} \underline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \geq 0 \\ \overline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \leq 0 \end{cases}$$

$$\text{if } g \geq 0$$

Two very useful properties

External additivity ✓

$$(\underline{P}_1 \otimes \underline{P}_2)(f(X_1) + h(X_2)) = \underline{P}_1(f(X_1)) + \underline{P}_2(h(X_2))$$

Factorisation ✓

$$(\underline{P}_1 \otimes \underline{P}_2)(g(X_1)h(X_2)) = \begin{cases} \underline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \geq 0 \\ \overline{P}_1(g(X_1))\underline{P}_2(h(X_2)) & \text{if } \underline{P}(h(X_2)) \leq 0 \end{cases}$$

if $g \geq 0$ is $\mathcal{B}_1$-measurable

Independent natural extension may not exist!

Independent natural extension always exists!

~~Walley~~ ↔ Williams ✓

~~value-independence~~ ↔ subset-independence ✓

Factorisation may not hold!

Factorisation always holds!

See you at the poster?