Algorithms for Multi-criteria optimization in Possibilistic Decision Trees

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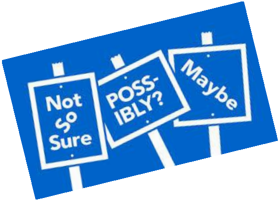
Context



Multi-criteria optimization in **Possibilistic Decision Trees**

Multi-criteria decision making (Without uncertainty)

MCDM aggregation

- Let Cr a set of criteria, $u_i(x)$ the utility of consequence x for each of them :
  - Disjunctive (max-based) aggregation : The satisfaction of the DM corresponds to the most satisfied criterion.

$$Agg^{max}(x) = \max_{i \in \mathcal{A}} u_i(x)$$

  - Conjunctive (min-based) aggregation : The satisfaction of the DM corresponds to the least satisfied criterion.

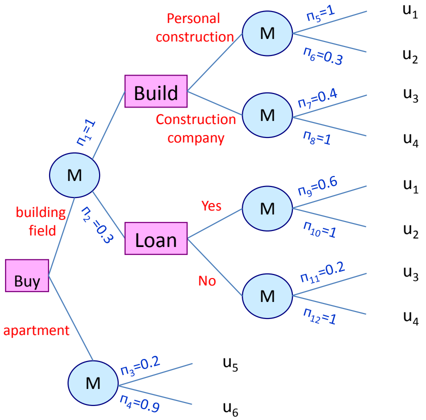
$$Agg^{min}(x) = \min_{i \in \mathcal{A}} u_i(x)$$

When the criteria do not have the same importance, a weight (importance degree) w_i is associated to each of them

Possibilistic decision trees [Sabbadin et al., 1998, Garcia and Sabbadin, 2006]

- Explicit modeling of sequential decision problems
- Graphical component : $\mathcal{D}$, $\mathcal{C}$ and $\mathcal{LN}$
- Numerical component : conditional possibilities and possibilistic utilities.

- Solving a DT → building a complete strategy
- decision rules : U_{opt} / U_{pes}



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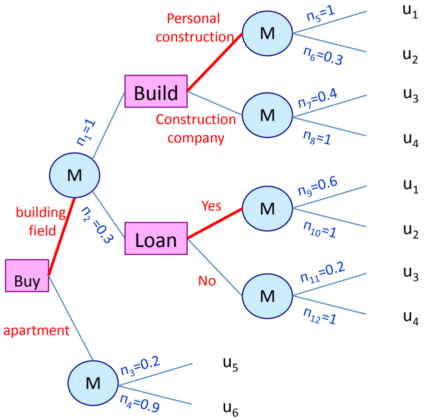

Possibilistic decision trees [Sabbadin et al., 1998, Garcia and Sabbadin, 2006]

δ^* is an optimal strategy w.r.t. a decision rule O, iff

$$\forall \delta \in \Delta, U_O(\delta^*) \succeq_O U_O(\delta)$$

Dynamic Programming

If the decision rule O satisfies

- Transitivity : $L \succ_O L'$ and $L' \succ_O L'' \quad \Rightarrow \quad L \succ_O L''$
- (weak) monotonicity : $L \succeq_O L' \quad \Rightarrow \quad \langle \alpha/L, \beta/L'' \rangle \succeq_O \langle \alpha/L', \beta/L'' \rangle$

then Dynamic programming may be performed.

Strategy optimization in possibilistic decision trees

Qualitative utilities U_{pes} and U_{opt} are transitive and monotonic.
$\Rightarrow$ Strategy optimization in polytime using Dynamic programming.

Multi-criteria qualitative decision problem

$\langle \mathcal{L}, \vec{w}, \vec{u} \rangle$

Given a set of consequence X, a set of criteria $Cr = \{1, \ldots, p\}$ we denote :

- $\mathcal{L}$: set of possibilistic lotteries,
- $\vec{w} \in [0, 1]^p$: weighting vector,
- $\vec{u} = \langle u_1, \ldots, u_p \rangle$: N vectors of p utility functions.

We have to consider

1. Optimistic or Pessimistic utility, (DM attitude w.r.t. uncertainty)
2. Min-based or max-based aggregation, (Multi-criteria aggregation)

Besides we have to...

Precise when the uncertainty have to be considered : before (*ex-ante*) or after (*ex-post*) performing multi-criteria aggregation.

Possibilistic Ex-ante utilities [Ben Amor et al., 2014]

Computing (pessimistic or optimistic) utility for each criterion j, then performs the multi-criteria aggregation.

- Pessimistic attitude

$$U_{ante}^{-min}(L) = \min_{j=1,p} \max((1 - w_j), \min_{x_i \in X} \max(u_j(x_i), 1 - L[x_i]))$$

$$U_{ante}^{-max}(L) = \max_{j=1,p} \min(w_j, \min_{x_i \in X} \max(u_j(x_i), 1 - L[x_i]))$$

- Optimistic attitude

$$U_{ante}^{+max}(L) = \max_{j=1,p} \min(w_j, \max_{x_i \in X} \min(u_j(x_i), L[x_i]))$$

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Pessimistic attitude — Optimistic attitude — Aggregation function

Possibilistic Ex-post utilities [Ben Amor et al., 2014]

Computing the (conjunctive or disjunctive) aggregated utility and then consider the uncertainty (mono-criterion problem).

- Pessimistic attitude

$$U_{post}^{-min}(L) = \min_{x_i \in X} \max(1 - L[x_i], \min_{j=1,p} \max((1 - w_j), u_j(x_i)))$$

$$U_{post}^{-max}(L) = \min_{x_i \in X} \max(1 - L[x_i], \max_{j=1,p} \min(w_j, u_j(x_i)))$$

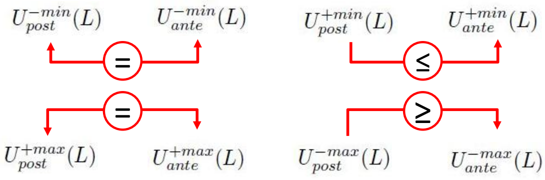
- Optimistic attitude

$$U_{post}^{+max}(L) = \max_{x_i \in X} \min(L[x_i], \max_{j=1,p} \min(w_j, u_j(x_i)))$$

$$U_{post}^{+min}(L) = \max_{x_i \in X} \min(L[x_i], \min_{j=1,p} \max((1 - w_j), u_j(x_i)))$$

Pessimistic attitude — Optimistic attitude — Aggregation function

Correlation between decision rules [Ben Amor et al., 2015]



Homogeneous utilities U^{+max} and U^{-min}

$$U_{ante}^{+max}(L) = U_{post}^{+max}(L) \text{ and } U_{ante}^{-min}(L) = U_{post}^{-min}(L).$$

$\hookrightarrow$ ex-ante and ex-post approaches provide the same results.

Heterogeneous utilities U^{+min} and U^{-max}

$$U_{ante}^{+min}(L) \neq U_{post}^{+min}(L) \text{ and } U_{ante}^{-max}(L) \neq U_{post}^{-max}(L).$$

$\hookrightarrow$ U^{+min} and U^{-max} suffer from timing effect.

Problematic and objectives

Solving sequential multi-criteria decision problem under qualitative uncertainty.

⇓

- Multi-criteria Possibilistic decision trees (leaves labeled by a utility vector)
- Algorithmic solution and optimal strategy (depends on decision rules properties)
- Experimental study (empirical comparison between algorithms)

Decision rules properties and algorithms

Decision Rule	*Transitivity*	*Weak Monotonicity*	*Algorithm*
U_{opt} / U_{pes}	x	x	**DynProg**

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Ex-post utilities: U^{-min}_{post} U^{+max}_{post} U^{-max}_{post} U^{+min}_{post}	x	x	**Aggregation + DynProg**

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Homogenous ex-ante utilities: U^{-min}_{ante} U^{+max}_{ante}	x	x	**Aggregation + DynProg**

Decision rules properties and algorithms

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Homogenous ex-ante utilities: U^{-min}_{ante} U^{+max}_{ante}	x	x	**Aggregation + DynProg**
U^{-max}_{ante}	x	-	**?**
U^{+min}_{ante}	x	-	**?**

U_{ante}^{-max} : Multi-Dynamic programming

$$U_{ante}^{-max}(L) = \max_{i=1,p} \min(w_j, U_j^-(L))$$

- A strategy δ_j^* that optimizes U^- for each criterion j.
- The one with the highest U_{ante}^{-max} is globally optimal (δ^*).

$\Rightarrow$ Multi-Dynamic Programming.

U_{ante}^{-max} : Multi-Dynamic programming (cont.)

Principle of the algorithm

- Consider the utility values of each criterion.
- Compute the pessimistic utility U_j^- by classical Dynamic programming.
- Return all optimal strategies and their corresponding values.
- Incorporate the weights of criteria.
- Choose the one that maximizes the U_{ante}^{-max}.

U^{+min}_{ante} : Branch and Bound

$$U^{+min}_{ante}(L) = \min_{j=1,p} \max(1 - w_j, U^{+}_{j}(L))$$

- Lack of monotonicity $\nrightarrow$ Dynamic programming may be suboptimal.
- Explicit enumeration $\nrightarrow$ Number of possible strategies is exponential.

$\Downarrow$

Implicit enumeration via BB

- The use of bounds combined with the value of the current best solution enables the algorithm to implicitly eliminate sub-trees.

U_{ante}^{+min} : Branch and Bound (Cont.)

Upper Bound

- Provides the best completion of the current strategy δ.
  - Builds a strategy δ_j for each criterion that maximizes the optimistic utility U_j^+ (using classical dynamic programming).
  - Selects the one that optimizes U_{ante}^{+min}.

- We can initialize the solution with strategy provided by dynamic programming.
  $\Rightarrow$ Reduce the algorithm's complexity

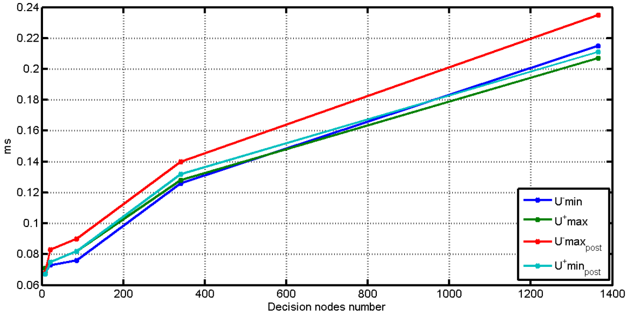
Experiments

Data

- Randomly generated binary possibilistic trees
  - Number of criteria : 3
  - Horizon (seq) : 2,3,4,5, 6 $\Rightarrow |D| = 5, 21, 85, 341, 1365$
  - Utility values, weights and conditional possibilities are randomly fired.

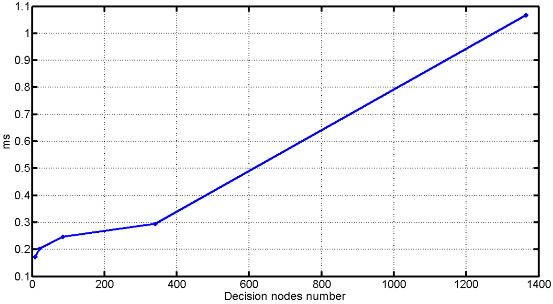
- Feasibility of the proposed algorithms.
- Quality of approximation(by dynamic programming).

Ex-post utilities : Dynamic Programming



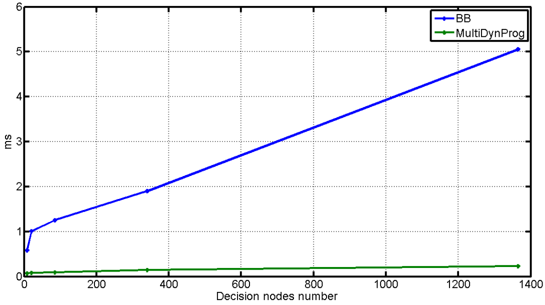
The CPU time increases linearly w.r.t. to the size of the tree.

U_{ante}^{-max} : Multi-Dynamic Programming



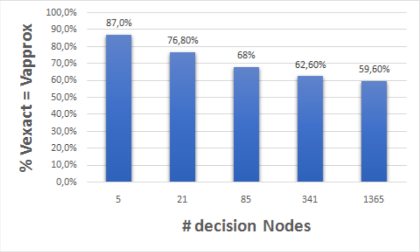
The CPU time of Multi-Dynamic Programming is very affordable.

U_{ante}^{+min} : Approximation by DynProg



Approximation using Dynamic programming is faster than BB.

U^{+min}_{ante} : Approximation by Ex-Ante DynProg

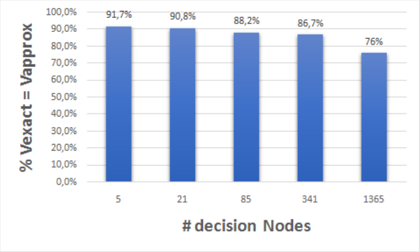


	Number of Decision Nodes				
	5	21	85	341	1365
$\frac{V_{Approx}}{V_{Exact}}$	0.97	0.95	0.94	0.93	0.91

- $\approx$ 70% of cases BB optimal strategy is also ex-ante optimal strategy.
- The closeness value $\frac{V_{Approx}}{V_{Exact}} > 0.9$.

$\Rightarrow$Dynamic programming can find a good strategy, even the optimal one.

U^{+min}_{ante} : Approximation by Ex-Post DynProg



	Number of Decision Nodes				
	5	21	85	341	1365
$\frac{V_{Approx}}{V_{Exact}}$	0.98	0.97	0.94	0.92	0.9

- $\approx$ 85% of cases BB optimal strategy is also ex-post optimal strategy.
- Very good closeness value.

$\Rightarrow$ ex-post dynamic programming is better than its ex-ante counterpart.

Conclusion & Future work

Conclusion

- Multi-criteria optimization in possibilistic decision trees.
- Dynamic programming algorithm and variants :
  ↪ Ex-post utilities collapse to classical optimization w.r.t. U^+ / U^-.
- Multi-dynamic Programming to optimize U_{ante}^{+min}.
- Branch and Bound algorithm for U_{ante}^{+min} (costly algorithm)
  ↪ Dynamic programming is a good approximation for U_{ante}^{+min}.

Future work

- Provide a complexity study
- Multi-criteria optimization for more sophisticated decision models (Influence diagrams, Markov decision models,...)
- Consider other (possibilistic or not) decision rules to express uncertainty.


thank you
danke
謝謝
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
bedankt
dziękuję
sukriya
kop khun krap
go raibh maith agat
mochchakkeram
tapadh leat
спасибо
рахмат
Баярлалаа
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dank je
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Possibilistic decision making

[Dubois and Prade, 1988a, Dubois and Prade, 1988b]

- Possibilistic distribution : $\pi : \Omega \to L = [0, 1]$
- **Normalization** : $\exists s \in S, \max_{s \in S} \pi(s) = 1$.

- Possibilistic lottery : $L = \langle \lambda_1/x_1, \ldots, \lambda_n/x_n \rangle$

λ_i : Possibility degree to obtain outcome x_i.

Qualitative utilities [Dubois and Prade, 1995, Dubois et al., 2001]

- Possibility and utility scales are commensurate
- Pessimistic utility : $U_{pes}(L) = \min_{x_i \in X} \max(1 - L[x_i]), u(x_i))$

  Evaluates to what extent it is certain (necessary) that L provides a good consequence.
- Optimistic utility : $U_{opt}(L) = \max_{x_i \in X} \min(L[x_i], u(x_i))$

  An optimistic counterpart of the pessimistic utility.

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