Complexity of Model Checking for Cardinality-based Belief Revision Operators

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Introduction

- **Belief revision, principles:**
  - Success
  - Consistency
  - Minimality of change

- **Two main approaches for belief revision:**
  - Semantic: models based.
  - Syntactic: formulas based

    Given *B* a belief base (finite set of formulas), μ new information (a formula),

    $B * \mu$ is the revised belief base.

    based on maximal subbases *B* consistent with μ.

Main objectives

- Presentation of different syntactic revision operators within a unified framework
- Introduction of two new cardinality-based operators
- Comparative study of the complexity of model checking for these operators, in different fragments of propositional logic.

Overview

1. Definition of syntactic belief revision operators
2. Complexity of Model Checking
3. Conclusion and perspectives

Syntactic Belief Revision operators

$B * \mu$ stems from $\mathcal{W}(B, \mu)$ the set of maximal subbsases of B consistent with μ.

- Maximality criteria for consistent belief subbases:
  - set inclusion/ cardinality.

- Strategies for exploiting the maximal consistent belief subbases:
  - (G) : all maximal subbases are equally plausible,

$$B *_G \mu = \bigvee_{B' \in \mathcal{W}(B,\mu)} \bigwedge (B' \cup \{\mu\})$$

  - (W) : "when in doubt, throw it out" (widtio), keep only beliefs that are not questioned

$$B *_W \mu = \bigwedge \bigcap_{B' \in \mathcal{W}(B,\mu)} (B' \cup \{\mu\})$$

Set-inclusion as maximality criterion

subbases of B consistent with μ maximal w.r.t. set inclusion

$$\mathcal{W}_{\subseteq}(B,\mu) = \{B_1 \subseteq B \mid \bigwedge B_1 \not\models \neg\mu \text{ and for all } B_2 \text{ such that } B_1 \subset B_2 \subseteq B, \bigwedge B_2 \models \neg\mu\}$$

Ginsberg operator $*_G$ and Widtio operator $*_{wid}$

$$B *_G \mu = \bigvee_{B' \in \mathcal{W}_{\subseteq}(B,\mu)} \bigwedge (B' \cup \{\mu\})$$

$$B *_{widtio} \mu = \bigwedge \bigcap_{B' \in \mathcal{W}_{\subseteq}(B,\mu)} (B' \cup \{\mu\})$$

Example

$B = \{a \to \neg b, b, b \to c, c \to \neg a, b \to d, d \to \neg a, \neg d \to c\}$ and $\mu = a$.

$$\begin{aligned}
\mathcal{W}_{\subseteq}(B, \mu) \quad = \quad & \{\{a \to \neg b, b \to c, c \to \neg a, b \to d, d \to \neg a\}, \\
& \{a \to \neg b, b \to c, c \to \neg a, b \to d, \neg d \to c\}, \\
& \{a \to \neg b, b \to c, b \to d, d \to \neg a, \neg d \to c\}, \\
& \{b, b \to c, b \to d, \neg d \to c\}, \\
& \{b, c \to \neg a, b \to d, \neg d \to c\}, \\
& \{b, b \to c, d \to \neg a, \neg d \to c\}, \\
& \{b, c \to \neg a, d \to \neg a\}\}
\end{aligned}$$

$$B *_G \mu = \bigvee_{B' \in \mathcal{W}_{\subseteq}(B,\mu)} \bigwedge (B' \cup \{\mu\}) \equiv a \wedge (b \to c)$$

$$B *_{\text{Widtio}} \mu = \bigwedge \bigcap_{B' \in \mathcal{W}_{\subseteq}(B,\mu)} (B' \cup \{\mu\}) \equiv a$$

Cardinality as maximality criterion

RSR stands for Removed Sets Revision

subbases of B consistent with μ maximal w.r.t. cardinality

$$\mathcal{W}_{card}(B,\mu) = \{B_1 \subseteq B \mid \bigwedge B_1 \not\models \neg\mu \text{ and for all } B_2 \subseteq B \text{ such that } |B_1| < |B_2|, \bigwedge B_2 \models \neg\mu\}$$

RSRG operator $*_{\mathrm{RSRG}}$ and RSRW operator $*_{\mathrm{RSRW}}$

$$B *_{\mathrm{RSRG}} \mu = \bigvee_{B' \in \mathcal{W}_{card}(B,\mu)} \bigwedge (B' \cup \{\mu\})$$

$$B *_{\mathrm{RSRW}} \mu = \bigwedge \bigcap_{B' \in \mathcal{W}_{card}(B,\mu)} (B' \cup \{\mu\})$$

Example

$B = \{a \to \neg b, b, b \to c, c \to \neg a, b \to d, d \to \neg a, \neg d \to c\}$, and $\mu = a$.

$$
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& \{a \to \neg b, b \to c, b \to d, d \to \neg a, \neg d \to c\}\}
\end{aligned}
$$

$$B *_{RSRG} \mu = \bigvee_{B' \in \mathcal{W}_{card}(B,\mu)} \bigwedge (B' \cup \{\mu\}) \equiv a \wedge \neg b \wedge (c \to \neg d)$$

$$B *_{RSRW} \mu = \bigwedge \bigcap_{B' \in \mathcal{W}_{card}(B,\mu)} (B' \cup \{\mu\}) \equiv a \wedge \neg b$$

Extension to stratified belief bases $B = (S_1, ..., S_n)$

$X \subseteq B$, $trace(X, B) = (|X \cap S_1|, ..., |X \cap S_n|)$.

maximality w.r.t. lexicographic order $\leq_{lex}$

$$\mathcal{W}_{cardlex}(B, \mu) = \{B_1 \subseteq B \mid \bigwedge B_1 \not\models \neg\mu \text{ and for all } B_2 \subseteq B \text{ s. t. } trace(B_1, B) <_{lex} trace(B_2, B), \bigwedge B_2 \models \neg\mu\}.$$

PRSRG operator $*_{\mathrm{RSRG}}$ and PRSRW operator $*_{\mathrm{RSRW}}$

$$B *_{\mathrm{PRSRG}} \mu = \bigvee_{B' \in \mathcal{W}_{cardlex}(B,\mu)} \bigwedge (B' \cup \{\mu\})$$

$$B *_{\mathrm{PRSRW}} \mu = \bigwedge \bigcap_{B' \in \mathcal{W}_{cardlex}(B,\mu)} (B' \cup \{\mu\}).$$

The operators we consider

Strategy	maximality criterion		
	set inclusion	cardinality	cardlex
G	Ginsberg ($*_G$)	RSRG ($*_{\mathrm{RSRG}}$)	PRSRG ($*_{\mathrm{PRSRG}}$)
W	Widtio ($*_{\mathrm{Widtio}}$)	RSRW ($*_{\mathrm{RSRW}}$)	PRSRW ($*_{\mathrm{PRSRW}}$)

The problem of Model Checking

Problem :	MODEL-CHECKING($*$)
Instance :	B a belief base, μ a formula, m an interpretation
Question :	$m \models B * \mu$?

The complexity of inference studied by *Nebel (1991), Eiter and Gottlob (1992), Nebel (1998), Cayrol et al. (1998).*

The complexity of Model Checking initiated in *Liberatore and Schaerf (2001)*, for operators based on a set-inclusion maximality criterion.

Complexity of MODEL-CHECKING($*$) for the G-strategy

Problem	Propositional Logic	Horn
MODEL-CHECKING($*_G$)	coNP-complete	P
MODEL-CHECKING($*_{RSRG}$)	coNP-complete	coNP-complete
MODEL-CHECKING($*_{PRSRG}$)	coNP-complete	coNP-complete

Idea of proof:
MAX-INDEPENDENT-SET reduces to the complementary of MODEL-CHECKING($*_{RSRG}$)

Complexity of MODEL-CHECKING($*$) for the W-strategy

Problem	Propositional Logic	Horn
MODEL-CHECKING($*_{\mathrm{Widtio}}$)	$\Sigma_2 P$-complete	NP-complete
MODEL-CHECKING($*_{RSRW}$)	$\Theta_2 P$-complete	$\Theta_2 P$-complete
MODEL-CHECKING($*_{PRSRW}$)	in $\Delta_2 P$, $\Theta_2 P$-hard	in $\Delta_2 P$, $\Theta_2 P$-hard

$\textsc{Model-Checking}(*_{\text{Widtio}})$ is NP-complete in the Horn fragment

- Membership: to prove that $m \models B *_{\text{Widtio}} \mu$, for every $\alpha \in B$ such that $m \not\models \alpha$, guess $B'_\alpha \subseteq B$ such that $B'_\alpha \cup \{\mu\}$ is consistent and $B'_\alpha \cup \{\mu\} \cup \{\alpha\}$ is inconsistent.
- Hardness:

Problem :	PQ-ABDUCTION
Instance :	a Horn formula φ, a set of variables $A = \{x_1, ..., x_n\}$ such that $A \subseteq Var(\varphi)$ and a variable $q \in Var(\varphi) \setminus A$.
Question :	Does there exist a set $E \subseteq \mathrm{Lit}(A)$ such that $\varphi \wedge \bigwedge E$ is satisfiable and $\varphi \wedge \bigwedge E \wedge \neg q$ is unsatisfiable?

  - PQ-ABDUCTION is NP-complete (Creignou and Zanuttini, 2006)
  - PQ-ABDUCTION $\leq$ $\textsc{Model-Checking}(*_{\text{Widtio}})$ when restricted to Horn formulas

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MODEL-CHECKING($*_{RSRW}$) is Θ_2P-complete

Idea of proof:

- Membership
  kmax: the maximal cardinality of subsets of B consistent with μ.
  - Check that $m \models \mu$
    - else, we have $m \not\models B *_{RSRW} \mu$.
  - Compute *kmax* (logarithmic number of calls to an NP-oracle).
  - For every $\alpha \in B$ such that $m \not\models \alpha$, does there exist $B'_\alpha \subseteq B \setminus \{\alpha\}$ consistent with μ such that $|B'_\alpha| = kmax$?
    - if yes, $m \models B *_{RSRW} \mu$.
    - else, $m \not\models B *_{RSRW} \mu$.

- Hardness: CARDMINSAT $\leq$MODEL-CHECKING($*_{RSRW}$).

Summary of the results

Operator	**Propositional logic**	**Horn**
Ginsberg	coNP-complete	P
Widtio	$\Sigma_2 P$-complete	NP-complete
RSRG	coNP-complete	coNP-complete
RSRW	$\Theta_2 P$-complete	$\Theta_2 P$-complete
PRSRG	coNP-complete	coNP-complete
PRSRW	in $\Delta_2 P$, $\Theta_2 P$-hard	in $\Delta_2 P$, $\Theta_2 P$-hard

Contribution and future work

- Syntactic belief revision operators presented within a unified framework
- Introduction of new operators based on maximal cardinality, $*_{RSRG}$, $*_{RSRW}$, $*_{PRSRG}$, $*_{PRSRW}$.
- Comparative study of the complexity of Model Checking in PL, as well as in Horn.

- Extend this study to other belief base revision strategies.
- Study the complexity of the inference problem for $*_{RSRG}$, $*_{RSRW}$, $*_{PRSRG}$, $*_{PRSRW}$.

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Thank you for your attention!