Reasoning in Description Logics with Typicalities and Probabilities of Exceptions

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DLs with Typicality and Probabilities

Outline

- Introduction to Description Logics
- DLs of Typicality
- Extensions with Probabilities
- Conclusions

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- Important formalisms of knowledge representation
- Two key advantages:
  - well-defined semantics based on first-order logic
  - good trade-off between expressivity and complexity
- at the base of languages for the semantic (e.g. OWL)

Knowledge bases

- Two components:
  - TBox=inclusion relations among concepts
  - ABox= instances of concepts and roles = properties and relations among individuals

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Description Logics

Introduction

- TBox $\Rrightarrow$ taxonomy of concepts
- need of representing prototypical properties and of reasoning about defeasible inheritance
- to handle defeasible inheritance needs the integration of some kind of nonmonotonic reasoning mechanism
  - DLs + MKNF
  - DLs + circumscription
  - DLs + default
- However, all these methods present some difficulties ...

Outline

DLs with typicality

- Non-monotonic extensions of Description Logics for reasoning about prototypical properties and inheritance with exceptions
  - Basic idea: to extend DLs with a typicality operator $\mathbf{T}$
  - $\mathbf{T}(C)$ singles out the "most normal" instances of the concept C
  - semantics of $\mathbf{T}$ defined by a set of postulates that are a restatement of Lehmann-Magidor axioms of rational logic $\mathbf{R}$

Basic notions

- A KB comprises assertions $\mathbf{T}(C) \sqsubseteq D$
- $\mathbf{T}(\mathit{Student}) \sqsubseteq \mathit{FacebookUsers}$ means "normally, students use Facebook"
- $\mathbf{T}$ is nonmonotonic
  - $C \sqsubseteq D$ does not imply $\mathbf{T}(C) \sqsubseteq \mathbf{T}(D)$

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The logic $\mathbf{ALC} + \mathbf{T}_{min}$

Example

$\mathbf{T}(Platypus) \sqsubseteq \neg \exists wears.Hat$

$\mathbf{T}(Platypus \sqcap SecretAgent) \sqsubseteq \exists wears.Hat$

Reasoning

- ABox:
  - $Platypus(perry)$
- Expected conclusions:
  - $\neg \exists wears.Hat(perry)$

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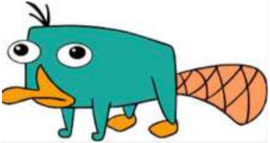

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Semantics

- $\mathcal{M} = \langle \Delta^{\mathcal{I}}, <, .^{\mathcal{I}} \rangle$
  - additional ingredient: preference relation among domain elements
  - $<$ is an irreflexive, transitive, modular and well-founded relation over $\Delta^{\mathcal{I}}$:
    - for all $S \subseteq \Delta^{\mathcal{I}}$, for all $x \in S$, either $x \in Min_<(S)$ or $\exists y \in Min_<(S)$ such that $y < x$
    - $Min_<(S) = \{u : u \in S \text{ and } \nexists z \in S \text{ s.t. } z < u\}$
  - Semantics of the $\mathbf{T}$ operator: $(\mathbf{T}(C))^{\mathcal{I}} = Min_<(C^{\mathcal{I}})$

Weakness of monotonic semantics

Logic $\mathcal{ALC} + \mathbf{T}$

- The operator $\mathbf{T}$ is nonmonotonic, but...
- The logic is monotonic
  - If KB $\models F$, then KB' $\models F$ for all KB' $\supseteq$ KB

Example

- in the KB of the previous slides:
  - if $Platypus(perry) \in$ ABox, we are not able to:

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    - assume that $\mathbf{T}(Platypus)(perry)$
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The nonmonotonic logic $\mathbf{ALC} + \mathbf{T}_{min}$

Rational closure

- Preference relation among models of a KB
  - $\mathcal{M}_1 < \mathcal{M}_2$ if $\mathcal{M}_1$ contains less exceptional (not minimal) elements
  - $\mathcal{M}$ minimal model of KB if there is no $\mathcal{M}'$ model of KB such that $\mathcal{M}' < \mathcal{M}$
- Minimal entailment
  - KB $\models_{min} F$ if F holds in all *minimal* models of KB
- Nonmonotonic logic
  - KB $\models_{min} F$ does not imply KB' $\models_{min} F$ with KB' $\supset$ KB
- Corresponds to a notion of rational closure of KB

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DLs + T and probabilities

Introduction

- $\mathcal{ALC} + \mathbf{T}^{\mathsf{P}}$: extension of $\mathcal{ALC}$ by typicality inclusions equipped by *probabilities of exceptionality*
- $\mathbf{T}(C) \sqsubseteq_p D$, where $p \in (0, 1)$
- intuitive meaning: typical Cs are also Ds with a probability p or normally, Cs are Ds and the probability of having exceptional Cs not being Ds is $1 - p$

Example

$$\mathbf{T}(\mathit{Student}) \sqsubseteq_{0.3} \mathit{SportLover}$$

$$\mathbf{T}(\mathit{Student}) \sqsubseteq_{0.9} \mathit{SocialNetworkUser}$$

- sport lovers and social network users are both typical properties of students
- probability of not having exceptions is 30% and 90%, respectively

DLs + T and probabilities

Probabilistic DLs

- $\mathcal{ALC} + \mathbf{T}^{\mathsf{P}}$ different from DLs with DISPONTE semantics
- probabilistic axioms $p\ :: C \sqsubseteq D$ used to capture uncertainty
  - Cs are Ds with probability p
- in $\mathcal{ALC} + \mathbf{T}^{\mathsf{P}}$ typical properties to concepts and to reason about probabilities of exceptions to those typicalities

DLs + T and probabilities

Basic idea

- extensions of an ABox containing only some of the "plausible" typicality assertions of the rational closure of KB
  - each extension represents a scenario having a specific probability
  - probability distribution among scenarios
  - nonmonotonic entailment restricted to extensions whose probabilities belong to a given and fixed range
  - reason about scenarios that are not necessarily the most probable

DLs + T and probabilities

Extensions of ABox

- typicality assumptions $\mathbf{T}(C_1)(a_1), \mathbf{T}(C_2)(a_2), \ldots, \mathbf{T}(C_n)(a_n)$ inferred from $\mathcal{ALC} + \mathbf{T}_{min}$
- extensions of ABox obtained by choosing *some* typicality assumptions
  - $\widetilde{\mathcal{A}_1} = \{\mathbf{T}(C_1)(a_1), \mathbf{T}(C_2)(a_2), \ldots, \mathbf{T}(C_n)(a_n)\}$
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- reasoning in the *monotonic* $\mathcal{ALC} + \mathbf{T}$ considering TBox and ABox extended with chosen assumptions

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Extensions of ABox and probabilities

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$\mathbf{T}(F) \sqsubseteq_{0.8} G$
$\mathbf{T}(C) \sqsubseteq_{0.5} H$

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$[0, 0.8, 0.105]$	$\widetilde{\mathcal{A}_3}$	$\mathbf{T}(C)(a)$	$\mathbf{T}(F)(a)$	$\mathbf{T}(C)(b)$
	$\vdots$			
$[0, 0, 0]$	$\widetilde{\mathcal{A}_8}$	$\mathbf{T}(C)(a)$	$\mathbf{T}(F)(a)$	$\mathbf{T}(C)(b)$

Extensions of ABox and probabilities

$\mathbf{T}(C) \sqsubseteq_{0.3} D$

$\mathbf{T}(C) \sqsubseteq_{0.7} E$

$\mathbf{T}(F) \sqsubseteq_{0.8} G$

$\mathbf{T}(C) \sqsubseteq_{0.5} H$

$\mathbf{T}(C)(a)$	$\mathbf{T}(F)(a)$	$\mathbf{T}(C)(b)$
0.105	0.8	0.105

$0.3 \times 0.7 \times 0.5$

$[0.105, 0.8, 0.105]$	$\widetilde{\mathcal{A}_1}$	$\mathbf{T}(C)(a)$	$\mathbf{T}(F)(a)$	$\mathbf{T}(C)(b)$	$\mathbb{P}_{\widetilde{\mathcal{A}_1}} = 0.105 \times 0.8 \times 0.105$
$[0.105, 0, 0]$	$\widetilde{\mathcal{A}_2}$	$\mathbf{T}(C)(a)$	$\mathbf{T}(F)(a)$	$\mathbf{T}(C)(b)$	$\mathbb{P}_{\widetilde{\mathcal{A}_2}} = 0.105 \times (1 - 0.8) \times (1 - 0.105)$
$[0, 0.8, 0.105]$	$\widetilde{\mathcal{A}_3}$	$\mathbf{T}(C)(a)$	$\mathbf{T}(F)(a)$	$\mathbf{T}(C)(b)$	$\mathbb{P}_{\widetilde{\mathcal{A}_3}} = (1 - 0.105) \times 0.8 \times 0.105$
	$\vdots$				
$[0, 0, 0]$	$\widetilde{\mathcal{A}_8}$	$\mathbf{T}(C)(a)$	$\mathbf{T}(F)(a)$	$\mathbf{T}(C)(b)$	$\mathbb{P}_{\widetilde{\mathcal{A}_8}} = (1 - 0.105) \times (1 - 0.8) \times (1 - 0.105)$

DLs + T and probabilities

Entailment

- Given KB$=(\mathcal{T}, \mathcal{A})$ and $p, q \in (0, 1]$
- $\mathcal{E} = \{\widetilde{A_1}, \widetilde{A_2}, \ldots, \widetilde{A_k}\}$ set of extensions of $\mathcal{A}$ whose probabilities are $p \leq \mathbb{P}_1 \leq q, p \leq \mathbb{P}_2 \leq q, \ldots, p \leq \mathbb{P}_k \leq q$
- $\mathcal{T}' = \{\mathbf{T}(C) \sqsubseteq D \mid \mathbf{T}(C) \sqsubseteq_r D \in \mathcal{T}\} \ \cup \ \{C \sqsubseteq D \in \mathcal{T}\}$
- KB $\models^{\langle p,q \rangle}_{\mathcal{ALC}+\mathbf{T}^{\mathrm{P}}} F$
  - if F is $C \sqsubseteq D$ or $\mathbf{T}(C) \sqsubseteq D$, if $(\mathcal{T}', \mathcal{A}) \models_{\mathcal{ALC}+\mathbf{T}_{min}} F$
  - if F is $C(a)$, if $(\mathcal{T}', \mathcal{A} \cup \widetilde{\mathcal{A}_i}) \models_{\mathcal{ALC}+\mathbf{T}} F$ for all $\widetilde{\mathcal{A}_i} \in \mathcal{E}$
- probability of F: $\mathbb{P}(F) = \sum_{i=1}^{k} \mathbb{P}_i$

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DLs + T and probabilities

TBox

$AtypicalDepressed \sqsubseteq Depressed$

$\mathbf{T}(Depressed) \sqsubseteq_{0.85} \neg\exists Symptom.MoodReactivity$

$\mathbf{T}(AtypicalDepressed) \sqsubseteq_{0.6} \exists Symptom.MoodReactivity$

$\mathbf{T}(ProstateCancerPatient) \sqsubseteq_{0.5} \exists Symptom.MoodReactivity$

$\mathbf{T}(ProstateCancerPatient) \sqsubseteq_{0.8} \exists Symptom.Nocturia$

Inferences

- $\mathbf{T}(Depressed \sqcap Tall) \sqsubseteq \neg\exists Symptom.MoodReactivity$ is entailed in $\mathcal{ALC} + \mathbf{T}^{\mathsf{P}}$
- if $\mathcal{A} = \{ProstateCancerPatient(jim), AtypicalDepressed(jim)\}$:
  - $\exists Symptom.MoodReactivity(jim)$ has probability 76%

DLs + T and probabilities

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Inferences

- $\mathbf{T}(Depressed \sqcap Tall) \sqsubseteq \neg \exists Symptom.MoodReactivity$ is entailed in $\mathcal{ALC} + \mathbf{T}^{\mathsf{P}}$
- if $\mathcal{A} = \{ProstateCancerPatient(jim), AtypicalDepressed(jim)\}$:
    - $\exists Symptom.MoodReactivity(jim)$ has probability 76%

Reasoning Procedure

procedure ENTAILMENT($(\mathcal{T}, \mathcal{A}), \mathcal{T}', F, \mathfrak{Tip}, p, q$)
  $\mathfrak{Tip}_{\mathcal{A}} \leftarrow \emptyset$ ▷ *build the set $\mathbb{S}$ of possible assumptions*
  for** each $C \in \mathfrak{Tip}$ **do
    for** each individual $a \in \mathcal{A}$ **do ▷ *Reasoning in $\mathcal{ALC} + \mathbf{T}_{\mathbf{R}}^{RaCl}$*
      ***if** $(\mathcal{T}', \mathcal{A}) \models_{\mathcal{ALC}+\mathbf{T}_{\mathbf{R}}^{RaCl}} \mathbf{T}(C)(a)$ **then** $\mathfrak{Tip}_{\mathcal{A}} \leftarrow \mathfrak{Tip}_{\mathcal{A}} \cup \{\mathbf{T}(C)(a)\}$*
  $\mathcal{P}_{\mathcal{A}} \leftarrow \emptyset$ ▷ *compute the probabilities of Definition 2 given $\mathcal{T}$ and $\mathfrak{Tip}_{\mathcal{A}}$*
  for** each $C \in \mathfrak{Tip}$ **do
    $\Pi_C \leftarrow 1$
    ***for** each $\mathbf{T}(C) \sqsubseteq_p D \in \mathcal{T}$ **do** $\Pi_C \leftarrow \Pi_C \times p$*
    $\mathcal{P}_{\mathcal{A}} \leftarrow \mathcal{P}_{\mathcal{A}} \cup \Pi_C$
  $\mathbb{S} \leftarrow$ build strings of possible assumptions as in Definition 3 given $\mathfrak{Tip}_{\mathcal{A}}$ and $\mathcal{P}_{\mathcal{A}}$
  $\mathcal{E} \leftarrow \emptyset$ ▷ *build extensions of $\mathcal{A}$*
  for** each $s_i \in \mathbb{S}$ **do
    build the extension $\widetilde{\mathcal{A}_i}$ corresponding to s_i and compute $\mathbb{P}_{\widetilde{\mathcal{A}_i}}$ as in Definition 4
    ***if** $p \leq \mathbb{P}_{\widetilde{\mathcal{A}_i}} \leq q$ **then** $\mathcal{E} \leftarrow \mathcal{E} \cup \widetilde{\mathcal{A}_i}$* ▷ *select extensions with probability in $\langle p, q\rangle$*
  for** each $\widetilde{\mathcal{A}_i} \in \mathcal{E}$ **do ▷ *query entailment in $\mathcal{ALC} + \mathbf{T}_{\mathbf{R}}$*
    ***if** $(\mathcal{T}', \mathcal{A} \cup \widetilde{\mathcal{A}_i}) \not\models_{\mathcal{ALC}+\mathbf{T}_{\mathbf{R}}} F$ **then return** $KB \not\models^{\langle p,q\rangle}_{\mathcal{ALC}+\mathbf{T}_{\mathbf{R}}^{\mathsf{P}}} F$*
  ***return** $KB \models^{\langle p,q\rangle}_{\mathcal{ALC}+\mathbf{T}_{\mathbf{R}}^{\mathsf{P}}} F$* ▷ *$F$ is entailed in all extensions*

DLs + T and probabilities

Results

- entailment restricted to extensions with a fixed probability / range of probabilities
- essentially inexpensive
  - entailment in in ExpTime as in the underlying $\mathcal{ALC}$

Beyond $\mathcal{ALC} + \mathbf{T}^{\mathsf{P}}$

Future works

- Combination of DLs with DISPONTE semantics with probability of exceptions
- Reasoning in real domains:
  - which range of probabilities?
- Implementation
- Extension to other DLs

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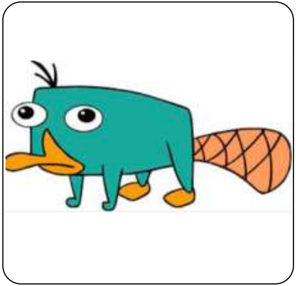
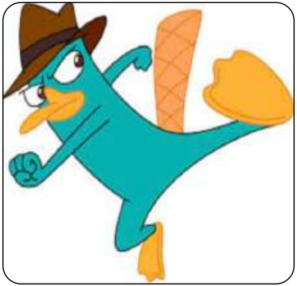
Any question?



Perry The Platypus
(aka Agent P)



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