Imprecise Swing Weighting

for Multi-Attribute Utility Elicitation Based on Partial Preferences

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Marmorkrebs

Origin unknown
First known individuals from pet trade 1990's
Can reproduce asexually
High reproduction rate
Damages ecosystems

Invasive Species Management

Aim

Eradicate invasive marmorkrebs recently observed in a lake

Decisions

(I) Do nothing
(II) Mechanical removal
(III) Drain system and remove individuals by hand
(IV) Drain system, dredge and sieve to remove individuals
(V) Decomposable biocide plus drainage
(VI) Increase pH plus drainage and removal by hand

Consequences

On Successful Eradication

Attribute	Worst (score 1)	Best (score 4)	Decision d I	II	III	IV	V	VI
Biotic impact	High	Low	4	4	3	3	2	1
Impact duration	Long	Short	4	4	3	3	1	2
Experience	Little	High	4	3	1	4	1	1
Feasibility	Difficult	Easy	4	4	2	3	1	2
Cost	High	Low	4	4	3	1	2	3

On Failed Eradication

Attribute	Worst (score 1)	Best (score 4)	Decision d I	II	III	IV	V	VI
Biotic impact	High	Low	1	1	1	1	1	1
Impact duration	Long	Short	1	1	1	1	1	1
Experience	Little	High	4	3	1	4	1	1
Feasibility	Difficult	Easy	4	4	2	3	1	2
Cost	High	Low	4	4	3	1	2	3

Application

Probability Bounds

Probability of Success	Decision d I	II	III	IV	V	VI
$\underline{p}_d$	0	0.05	0.3	0.4	1.0	0.7
$\overline{p}_d$	0	0.25	0.5	0.7	1.0	0.8

Utility Weight Bounds

1. Rewards: vary status quo per attribute
2. Bounding: compare these as per method by expert elicitation

Details in the paper!

Interval Dominance

Decision	Lower Utility	Upper Utility
I	0.25	0.37
II	0.23	0.47
III	0.18	0.31
IV	0.38	0.57
V	0.14	0.17
VI	0.11	0.17

Imprecise Swing Weighting

Assumptions

(i) Preferences form a preorder $\succeq$ on $L(\mathcal{R})$ and can be represented through a *set* $\mathcal{U}$ of utility functions $U\colon L(\mathcal{R}) \to \mathbb{R}$:

$$\ell_1 \succeq \ell_2 \iff \forall U \in \mathcal{U}\colon U(\ell_1) \geq U(\ell_2)$$

for all ℓ_1 and $\ell_2 \in L(\mathcal{R})$

(ii) Marginal utilities $U_i(a_i)$ precisely known

(iii) Joint utility has additive form:

$$U(a_1, \dots, a_n) = \sum_{i=1}^n k_i U_i(a_i)$$

How to identify the weights $(k_1, \dots, k_n)$?

Method

(i) Consider *any* joint rewards $r_0, \dots, r_n$ for which we have that

$$r_0 \preceq r_j \preceq r_n$$

(ii) Find largest $\underline{\alpha}_j$ and smallest $\overline{\alpha}_j$ so that

$$(1-\underline{\alpha}_j) r_0 \oplus \underline{\alpha}_j r_n \preceq r_j \preceq (1-\overline{\alpha}_j) r_0 \oplus \overline{\alpha}_j r_n$$

(iii) Derive set of linear inequalities on weights $(k_1, \dots, k_n)$ by imposing

$$(1-\underline{\alpha}_j) U(r_0) + \underline{\alpha}_j U(r_n) \leq U(r_j) \leq (1-\overline{\alpha}_j) U(r_0) + \overline{\alpha}_j U(r_n)$$

Main Result: Consistency & Uniqueness

Desirata

1. Solution for all possible choices of $0 \leq \underline{\alpha}_j \leq \overline{\alpha}_j \leq 1$
2. Unique solution when $\underline{\alpha}_j = \overline{\alpha}_j$ for all j

Precise Case

Consider precise case for any $\alpha_1, \dots, \alpha_{n-1} \in [0, 1]$. Assume that u_0 is constant, and that the vectors $(u_1, \dots, u_{n-1}, 1)$ are linearly independent. Let λ_j be the coefficients that decompose u_n as a linear combination of $(u_1, \dots, u_{n-1}, 1)$, i.e.

$$u_n = \lambda_n + \sum_{j=1}^{n-1} \lambda_j u_j \qquad (1)$$

Then the elicitation problem has a unique solution if and only if

$$\sum_{j=1}^{n-1} \alpha_j \lambda_j \neq 1 \qquad (2)$$

Imprecise Case

When $\lambda_1 \leq 0, \dots, \lambda_{n-1} \leq 0$, then desirata are always satisfied.

Contributions & Conclusions

– generalisation of the swing weighting method for eliciting multi-attribute utility functions allowing partial preferences; also see earlier work [6, 8, 4, 7, 3]
– enable practically handling new very wide range of problems where preference can only be partially specified
– novel, strong, and very general consistency result
– demonstration of method on a practical example with imprecision in both utility and probability

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Background image: https://www.pinterest.com/evigsvartvann/crustaceans-astacidea-lobsters-and-crayfish/